Comparison of Flow Characteristics on the Both Sides of Porous Preform in Resin Transfer Molding

Ming-Hsieh Hsien¹, Chih-Yuan Chang²,* and Lih-Wu Hourng¹

¹Department of Mechanical Engineering, National Central University, Chung-Li, Taiwan 32054 and ²Department of Mechanical and Automation Engineering, Kao Yuan University, Lu-Chu, Taiwan 82101

Abstract: A two-dimensional flow model has been developed to investigate the resin impregnation for the unidirectional fabrics with small gaps and small fiber bundles. The effect of various ratios of gap size to bundle width (h/D) on the micro-flow interaction is analyzed. The liquid flow within the fiber bundles is modeled by the Brinkman’s equation and the liquid motion in the gap is a simplified creeping flow. Results show that the liquid fronts in the gap move faster than that within the bundle and it is the potential for void entrapment within the bundle. The possibility for void entrapment can be reduced by using a low ratio of h/D or a high bundle permeability. The pressure interaction is trivial for a small gap. Contrarily, the small bundle has a strong influence on the pressure distribution. A boundary layer is significant near the permeable interface. The thickness of the boundary layer increases with large gap size and low bundle permeability. Thus, Brinkman’s equation must be applied for the unidirectional fabric having both a high ratio of h/D and a low bundle permeability.

Keywords: Resin Transfer Molding, Brinkman’s equation, Creeping flow.

1. INTRODUCTION

Resin transfer molding (RTM) is a well known processing method for polymer composites in which a liquid polymeric resin is impregnated through a fibrous preform in a closed mold. Due to the complex micro-structures of the fabrics, the behavior of the flow front tends to be complicated. The flow-induced voids are difficult to be avoided during the filling process.

The void content within RTM parts is responsible for a serious degradation of mechanical strength [1,2]. Several mechanisms for void formation include: flow-induced entrapment, and volatile material released by the resin or fiber during mold filling or curing. Many experimental analyses on the flow-induced void formation have been made [3-10]. Using the visualization experiments, Patel et al. [3] observed that the flow between the fiber bundles led the flow within the fiber bundles at a high flow rate; contrarily the flow front within the fiber bundles led the flow front between the fiber bundles at a low flow rate. Lundstrom and Gebart [4] and Chen et al. [5] reported two kinds of voids by image analysis: long cylindrical voids within the fiber bundles and large spherical voids in the space between the fiber bundles. Patel et al. [6] observed that the fingering took place at flow front because the permeabilities in the fiber tow and in the gap between the fiber tows were different. Rohatgi et al. [7] reported that the micro-voids were formed at capillary number greater than $10^{-3}$ for the axial flow and the micro-voids were formed at a low capillary number about $10^{-4}$ for the transverse flow. Matsuzaki et al. [8] developed an in situ method for measuring the void content during resin impregnation by combining image analysis with a technique that visualized the resin flow and void formation. Caba and Koch [9] utilized a design of experiment method to analyze the void formation mechanism and showed that most voids were flow-induced. Matsuzaki [10] developed a mathematical model of inter-bundle void formation in a plain-woven fabric.

Based on the concept that the permeability in the fiber bundle was much less than that in the gap, the potential for void formation was investigated [11-14]. Chan and Morgan [11] predicted that the void formation localized at the resin front region. Chang [12] reported that the amount of penetrating resin through the permeable interface to the fiber bundles was little during resin injection. Chen et al. [13] found that the quantities of the voids decreased with increasing flow rates at a higher filling rate or capillary number. Shih et al. [14] applied the random walk theory to investigate the void formation. They reported that the initial void could be reduced efficiently at the capillary number lower than $1.38 \times 10^{-3}$. Patel and Lee [15] developed a phenomenological model based on the multi-phase Darcy’s law and found that elimination of micro-voids from the fiber tows was very difficult. Pillai and Advani [16] utilized the unsaturated flow model in woven fiber preforms and reported that the pore volume ratio played an important role in such flows. Kang et al. [17]...
developed a model in which the effects of resin velocity and capillary pressure were described by the capillary number. With proper calibration, the model could predict the size and content of voids within fiber tows as well as between them. Gourichon et al. [18] proposed a new numerical procedure implemented within liquid injection molding simulation. They introduced the concept of the critical pressure for void mobilization and showed that void content should be considered on both macro- and micro-scale.

During the filling stage of RTM, the complex behavior of the flow front leads to the potential for void formation which degrades the quality of the composite components. Although several literatures are accessible concerning flow-induced voids, they often utilize Darcy’s law to model the flow interaction. This study is focused on the effect of small gap and small bundle on the micro-flow interaction for the unidirectional fabrics. The sum of the gap width (h) and bundle width (D) is set to $20\sqrt{K_x}$, where $K_x$ is the bundle permeability in the x direction. The flow in the gaps is a simplified creeping motion and the flow within the bundles is modeled by the Brinkman’s equation. The influences of the viscous stress on the liquid flow are also discussed.

2. THEORY

The fiber filaments are packed to form a small fiber bundle for the unidirectional fabrics. Hence two patterns of resin flow can be observed in the filling process. One is the resin flow in the gap; the other is the resin flow within the bundle. The flow front in the gap advances ahead of that within the fiber bundle since the interstitial space within the fiber bundle is much smaller than the space in the gap. Once the flow front in the gap encounters the transversal stitch, there is a transverse flow that leads to the trapping of air within the bundle.

For simplification, the resin flow is assumed to be parallel to the fiber axis. The gap is a fiber free region and the bundle is treated as a porous medium. Figure 1 provides a sketch of interaction between two resin flows.

For a slow flow in the gap, the continuity equation of the flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

where $u$ and $v$ are the liquid velocities in the x and y directions, respectively. The momentum equation is simplified due to $\frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 v}{\partial y^2} \gg \frac{\partial^2 v}{\partial x^2}$ by the order of magnitude analysis.

$$\frac{\partial P}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}$$

where P represents the liquid pressure and $\mu$ is the liquid viscosity.

For a small fiber bundle, the liquid flow can not be properly modeled using Darcy’s law because the reduced order of Darcy’s equation does not allow for the continuity of velocity at the permeable interface. The momentum equation of the flow is thus described by Brinkman’s equation as

$$\frac{\partial P}{\partial x} = -\frac{\mu}{K_x} u + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

Figure 1: The interaction between the flow in the gaps and the flow within the bundles.
\[
\frac{\partial P}{\partial y} = -\frac{\mu}{K_y} v + \tilde{\mu} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3b)
\]

where \( K \) is the bundle permeability, \( \tilde{\mu} \) denotes an effective viscosity that can differ from liquid viscosity. Here, \( \mu/\tilde{\mu} \) is set to unity according to the previous investigations [19,20].

The pressure equation can be obtained by combining 
\[
\frac{\partial}{\partial x} \left( \text{Equation (3a)} \right) \quad \text{with} \quad \frac{\partial}{\partial y} \left( \text{Equation (3b)} \right)
\]
and utilizing continuity equation.

\[
\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = -\left( \frac{\mu}{K_x} \frac{\partial u}{\partial x} + \frac{\mu}{K_y} \frac{\partial v}{\partial y} \right) \quad (4)
\]

Only half of the flow region needs to be concerned due to symmetric conditions along the center lines of gap and bundle. The boundary conditions are

\[
\frac{\partial u}{\partial y} = 0, \quad \frac{\partial v}{\partial y} = 0 \quad \text{along the center lines of gap and fiber bundle} \quad (5a)
\]
\[
P = P_{amb} \quad \text{along the flow front} \quad (5b)
\]
\[
(t)_{\text{gap}} = (t)_{\text{fiber}} \quad \text{along the permeable interface} \quad (5c)
\]
\[
Q_{in} = \text{constant} \quad \text{along the inlet} \quad (5d)
\]

where \( P_{amb} \) is the ambient pressure which is set to 100 kPa. \( \tilde{t} \) is viscous stress, and thus Equation (5c) is the continuous stress at the permeable interface besides the essential continuities of liquid pressure and liquid velocity. Equation (5d) is the inlet condition utilizing a constant volumetric flow rate \( (Q_{in}) \). It can be determined by integrating the liquid velocity across the area.

\[
Q_{in} = \int \vec{V} \cdot d\vec{A} \quad (6)
\]

where \( \vec{V} \) can be the inlet velocity or the front velocity.

3. NUMERICAL PROCEDURE

3.1. Grid Generation

One of the main difficulties of simulating the filling process is the numerical treatment of moving and irregular liquid front. An orthogonal mesh is generally desired to reduce the truncation error induced from the boundaries. By applying the body-fitted grid generation technique, the irregular physical domain \( (x-y) \) plane can be transformed to the rectangular computational domain \( (\xi-\eta) \) plane. It provides many advantages such as uniformly spaced grids, easily application of surface boundary conditions and modular technique, etc.

In the body-fitted method, a set of elliptic partial differential equations is used as follows [21]

\[
\xi_{xx} + \xi_{yy} = \Omega(\xi, \eta)(\xi_x^2 + \xi_y^2) \quad (7a)
\]
\[
\eta_{xx} + \eta_{yy} = \Psi(\xi, \eta)(\eta_x^2 + \eta_y^2) \quad (7b)
\]

where \( \Omega \) and \( \Psi \) are the control functions. Interchanging the dependent variables \( (\xi, \eta) \) with the independent variables \( (x,y) \) in the above equations yields

\[
a(x_{xx} + \Omega x_x) - 2b x_{xy} + c(x_{yy} + \Psi x_y) = 0 \quad (8a)
\]
\[
a(y_{xx} + \Omega y_x) - 2b y_{xy} + c(y_{yy} + \Psi y_y) = 0 \quad (8b)
\]

where \( a = x_y^2 + y_y^2, \ b = x_x x_y + y_x y_y \) and \( c = x_x^2 + y_y^2 \). \( \xi \) and \( \eta \) are taken as coordinates in the computational domain. The control functions, based on the assumptions that the curvature along the geometric boundaries is locally zero and the grid lines are orthogonal each other in the vicinity of geometric boundaries, are determined by

\[
\Omega = -\left( \frac{x_{xx} x_x + y_{xx} x_y}{x_x^2 + y_x^2} \right) \quad (9a)
\]
\[
\Psi = -\left( \frac{x_{yy} y_y + y_{yy} y_y}{x_y^2 + y_y^2} \right) \quad (9b)
\]

along the geometric boundaries. For the interior points, the control function is interpolated from that on the geometric boundaries. The governing equations are all transformed to and solved on the computational domain instead of on the physical domain.

Due to expansion of the flow region, the nodal coordinates need to be updated at each time step in this numerical scheme. It causes a time-consuming computation particular for using a large number of nodes. For the sake of enhancing the computational efficiency, an interpolation of curve of cubic spline is used to approach the shape of the flow front. The method offers the advantage of the piece-wise continuity and the avoidance of the singularity of the flow front.

3.2. Numerical Procedure

In the present scheme, the flow field is solved by the SIMPLER algorithm. The numerical procedure is stated as below.

1. Guess the position of the flow front in the \( n \)-th time step by the formula of \( \vec{R}_{n-1} + dt \vec{V}_{n-1} \cdot \vec{R}_{n-1} \) and
\( \bar{V}_{n-1} \) are the front position and velocity in the \((n-1)\)-th time step, respectively. \( dt \) is the time interval.

2. Guess the liquid velocity along the permeable interface.

3. Solve the equations (3)~(5) by the SIMPLER algorithm.

4. Repeat step 2 to step 4 until the liquid velocity along the permeable interface converges to the correct value.

5. Update the position of the flow front by the formula of
\[ R_{n-1} + dt \times \frac{\left(\bar{V}_{n-1} + \bar{V}_n\right)}{2} . \]
Repeat steps 2 to 5 until the flow front converges to the correct position. \( \bar{V}_n \) is the front velocity in the \(n\)-th time step.

6. Repeat step 1 to step 6 to predict the flow front in the \((n+1)\)-th time step.

For SIMPLER algorithm, the sequence of operations is described in reference [22]. A staggered grid is used to avoid odd-even decoupling between the pressure and velocity. It ensures the presence of real non-zero pressure gradient across the nodes in any condition, even in the case of a checkered grid. The details of the numerical procedure are described by the flow chart as shown in Figure 2.

4. RESULTS AND DISCUSSION

Figure 3 shows the flow front at various filling times for \( Q_{in} = 1.8 \times 10^{-8} \) m\(^3\)/s and \( \frac{h}{D} = 0.25 \). The viscosity of the liquid is 1 Pa-s. Dimensionless coordinates are used such as \( X^* = \frac{x}{\sqrt{K_x}} \) and \( Y^* = \frac{y}{\sqrt{K_x}} \). As can be expected, the smooth flow fronts can be completely modeled by using the Brinkman’s equation. The flow fronts in the gap always move faster than that within the fiber bundles since the viscous resistance in the gap is much less than that within the fiber bundle. It leads to the potential for void entrapment within the fiber bundles. The mechanism of the void formation has been proven by the previous experiments [3,6]. The degree of the front lead-lag can be reduced by increasing the flow friction in the gap or decreasing the flow resistance within the bundle. This is to say, the possibility for void entrapment is low for the case using a low ratio of \( \frac{h}{D} \) or a large bundle permeability. If the flow front does not encounter the transversal stitches, an approximately steady shape of the flow front can be reached after the filling time of \( 9 \times 10^2 \) second.

Figure 2: Flow chart of the numerical procedure.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{flow_chart.png}
\caption{Flow chart of the numerical procedure.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{flow_front.png}
\caption{Flow front at various filling times.}
\end{figure}

Figure 4 shows the average inlet pressure in the gap and within the fiber bundle for \( Q_{in} = 1.8 \times 10^{-8} \) m\(^3\)/s and \( \frac{h}{D} = 0.25 \). The inlet pressure within the fiber bundle is higher than that in the gap since the flow resistance offered by the fiber bundle is larger. During the filling
process, an approximately linear increase of the inlet pressure is present since total friction between the liquid and solid fiber increases as the flow region expands. However, the increase of the inlet pressure within the bundle is rapid in the early filling stage. It may be because the shape of the flow front does not reach steady state yet.

Figure 4: Variation of inlet pressure during the filling process.

Figure 5 shows the pressure distribution for $\frac{h}{D} = 0.053$ and $\frac{h}{D} = 4$ at the filling time of $6 \times 10^{-3}$ second. Evidently the liquid pressure gradually decreases with increasing liquid travelling distance due to viscous frictions. For low ratio of $\frac{h}{D}$, the total friction is large and thus the maximum pressure in Figure 5a is high relative to that in Figure 5b. The liquid pressure is approximately uniform for the case of $\frac{h}{D} = 0.053$ and it implies that the effect of small gap on the pressure interaction is trivial. This is because the gap size is approximately equal to the effective pore diameter of the fiber bundle. On the contrary, for the case of $\frac{h}{D} = 4$, the effect of small bundle on the liquid flow is significant because the fiber bundle offers a high resistance for liquid flow.

The difference between Darcy’s law and Brinkman’s equation is the viscous stress term. The viscous stress is important in a small region near the permeable interface of the fiber bundles. The region is defined as a boundary layer [19]. Figure 6 shows the relationship between dimensionless boundary layer thickness $\delta (\sqrt[5]{K_x})$, ratio $\frac{h}{D}$ and bundle permeability ($K_x$).

Figure 5: Pressure distribution at the filling time of $6 \times 10^{-3}$ second. (a) $\frac{h}{D} = 0.053$ (b) $\frac{h}{D} = 4$.

Through the numerical analysis, the boundary layer thickness is several times the size of $K_x^{0.5}$. The result agrees with the theoretical analyses [19,20]. The thickness of the boundary layer increases with higher ratio of $\frac{h}{D}$ and lower bundle permeability due to high liquid velocity difference between the gap and the bundle. A larger gap or a low bundle permeability can lead to a high liquid velocity difference. It indicates that...
Brinkman’s equation must be applied for the unidirectional fabric having both a high ratio of $\frac{h}{D}$ and low bundle permeability since the boundary layer will grow, meet and permeate the entire flow.

![Figure 6: Boundary layer thickness versus h/D and bundle permeability.](image)

5. CONCLUSIONS

A two-dimensional flow model is developed to investigate the interaction between the flow in the gaps and the flow within the bundles. This study is focused on the liquid flow for the unidirectional fabrics with small gaps and small bundles. The flow behavior is solved by the SIMPLER algorithm and an interpolation of curve of cubic spline is employed to approach the shape of the flow front. Results show that the smooth flow fronts can be completely modeled by using the Brinkman’s equation and simplified creeping flow. The potential for void formation within the bundle can be reduced by using a low ratio of h/D or a high bundle permeability. The pressure interaction is trivial for the ratio of $\frac{h}{D} = 0.053$, whereas the small bundle has a strong influence on the pressure distribution. The thickness of the boundary layer increases with large gap size and low bundle permeability. Thus, Brinkman’s equation must be applied for the unidirectional fabric having both a high ratio of h/D and a low bundle permeability.

ACKNOWLEDGMENT

This work was supported by the National Science Council of Republic of China [grant number NSC 89-2212-E-244-009].

NOMENCLATURE

- $D$ = width of the fiber bundle
- $h$ = gap size
- $K$ = permeability
- $P$ = pressure
- $Q_{in}$ = volumetric flow rate
- $u, v$ = velocity components in the physical domain
- $x, y$ = coordinate components in the physical domain
- $\xi, \eta$ = coordinate components in the computational domain
- $\delta$ = boundary layer thickness
- $\Omega, \Psi$ = control functions in the body fitted method
- $\mu$ = viscosity of resin

Superscript
- * = dimensionless parameters

REFERENCES


